Assignment for class 12

Chapter 4. Determinants (Introduction, MINORS AND COFACTORS)

General Direction For Candidates : Whatever be the notes provided , everything must **be** copied in the maths copy and then do the homework in the same copy.

Determinants(Introduction)

- A determinants is defined as (mapping) function from set of a square matrices to the set of real numbers.
- Every square matrix A is associated with a number , called the determinant denoted by det (A) or $|A|$ or Δ
- If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A|$ = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = det (A)
- Only square matrices have determinants
- For matrix A , $|A|$ is read as determinant of A and not modulus of A

Types of Determinants :

1. First order Determinant : Let A= $[a]$ be the matrix of order 1 , then determinant of A is defined to be equal to a ie det (A) =$|A|$ = a
2. Second order Determinant :
   - Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be matrix of order 2x2

   = det (A) = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ = $a_{11.}a_{22} - a_{21.}a_{12}$

   - Example ;

     If A =$\begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$ Then det (A) = 2x1-5x6= 2- 30= -28

     Q,8 (i) If xƐN and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, **Find the value of x**.

     Solution: (x+3)(2x) -(-3x)(-2) =8

     $\Rightarrow$ $2x^2$ +6x-6x =8

     $\Rightarrow$ $2x^2$ =8

     $\Rightarrow$ x=$\pm$ 2 , $\therefore$ x=2, x$\neq$ -2 because xƐN

     HOMEWORK: Q1. (i). (ii) Q2.(i) 3(ii) 5(i), 7(ii) ,8 (ii), (iii)
3. Third order determinant :
   - Its determinant can be found by expressing it in terms of second order determinant

- Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be matrix of order 3x3

  The following method is the basic method and is done by expansion around Row 1 (for detail explanation refer to video uploaded on you tube) . This will be more clear when minor and cofactor will be explained further

  Then det (A) = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ = $a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

  det(A) = $a_{11}(a_{22}.a_{33} - a_{32}.a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

  - **Example ; Exercise 4.1 Q.11. (i)** Let A = $\begin{bmatrix} 2 & 4 & 1 \\ 8 & 5 & 2 \\ -1 & 3 & 7 \end{bmatrix}$

    Then det (A) = $2\begin{vmatrix} 5 & 2 \\ 3 & 7 \end{vmatrix} - 4\begin{vmatrix} 8 & 2 \\ -1 & 7 \end{vmatrix} + 1\begin{vmatrix} 8 & 5 \\ -1 & 3 \end{vmatrix}$

    =2[(5)x(7)-(3)x(2)] - 4[(8)x(7)-(-1)x(2)] + 1[(8)(3) -(-1)x(5)]

    =2[(35)-(6)] -4[(56)-(-2)]+1[(24)-(-5)]

    =2x29-4x58+1x19

    =58-232+29

    =87-232

    =-145

    Please Note :

  - Expanding a determinant along any row or column gives the same value.
  - For easier calculation , we shall expand the determinant along that row or column which contains maximum number of zeros
  - In general , if A=KB where A and B are square matrices of order n , then $|A| = k^n |B|$, where n =1,2,3
  - This method doesn't work for determinant of order greater than 3

HOMEWORK

Exercise 4.1 (Understanding ISC Mathematics Class XII Volume 1)

Question No 10, 11(ii)(iii) ,12

MINOR

- If row and column containing the element a_{11} (ie 1st row and 1st column) are removed , we get second order determinant which is called Minor of element a_{11}
- Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies
- Minor of an element is denoted by M_{ij}
- Minor of an element of a determinant of order n (n≥2) is a determinant of order n-1
- Example : Find Minor of the element 5 in the determinant A given below

  $\det(A) = \begin{vmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix}$

  Since 5 lies in the second row and second column , its Minor obtained by deleting R_2 and C_2 is given by :

  $M_{22} = \begin{vmatrix} 9 & 7 \\ 3 & 1 \end{vmatrix} = 9 \times 1 - 3 \times 7 = 9 - 21 = -12$

COFACTOR

- If the minor are multiplied by proper signs we get cofactors
- The cofactor of the element a_{ij} is denoted by A_{ij}
- $\mathbf{A_{ij}=(-1)^{i+j}M_{ij}}$

  Exercise 4.1 (M.L. AGARWAL , UNDERSTANDING MATHEMATICS)

  Q4(i) Find the cofactor of a_{12} in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

  solution:

  $a_{12}=-3$, Minor of $a_{12} = M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$ (obtained by deleting1st row and 2nd column

  Cofactor of $a_{12} = A_{12} = (-1)^{1+2} M_{12} = (-1)^3 \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = (-1)(-42-4)=(-1)(-46)=46$

HOMEWORK

Exercise 4.1 Question No 4 (ii), (iii)

Question 13. using cofactors of elements of 2nd row , evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Solution:

Formulae to be used (sum of, the product of the element of 2nd row and its cofactor)

$a_{21}.A_{21} + a_{22}.A_{22}\ a_{23}.A_{23}$

$$= -2\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} + 0\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$$

=-2(9-16) +0(15-8) -1 (10 -3)

=-2(-7) +0(7) -1 (7)

= 14 +0 -7

=7

HOMEWORK:

Given $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Find i) a_{31} , ii)a_{32} iii) a_{33} iv)A_{31} v) A_{32} vi)A_{33} vii) $a_{31}\ A_{31} + a_{32}\ A_{32} + a_{33}\ A_{33}$

viii) Similarly find $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$